Non-local Lee-Wick modes in the fermionic Myers-Pospelov model

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General solutions and dispersion relations are given explicitly in the Lorentz invariance violating fermionic sector of the Myers and Pospelov theory. We quantize the theory and identify Lee-Wick modes due to the higher time-derivative terms. We analyze the non local character of these modes and discuss the loss of microcausality.

PACS numbers: 11.30.Cp, 11.10.Lm, 14.80.-j

I. INTRODUCTION

The need for a more fundamental theory at high energies has been justified in many contexts. Divergences in quantum field theory, singularities in gravity and having a unified quantum framework for all forces, are some of them. A consequence arising from this consideration, which has been extensively studied, is the possibility of having Lorentz invariance violation given in the form of effective corrections [1]. This idea naturally leads to new extensions of the standard model and modified dispersion relations for particles which may to be detected in sensitive experiments [2].

In this contexts the Myers-Pospelov theory is a model that introduces Lorentz invariance violation through dimension five operators [3, 4]. The breakdown of Lorentz symmetry that is included in the scalar, fermion and gauge sectors is characterized by an external timelike four-vector n_μ defining a preferred reference frame. Experimental bounds for this model have been studied in several phenomena, such as synchrotron radiation [5], gamma ray bursts [6], neutrino physics [7], radiative corrections [8]. generic backgrounds [9, 10], and others [11].

In recent years and from a different approach, theories with higher time-derivatives have been proposed as an extension of the standard model of particles [12]. One of the main motivations is that these theories soften the ultraviolet behavior of the quantum field theory, and hence problems like the hierarchy puzzle seems to be solved. Despite these theories contain negative norm states [13] the theoretical consistency were established many years ago [14]. Although unitarity is maintained, the price to pay is the lost of microcausality [15].

The lost of microcausality and higher time derivatives are ingredients present in Myers-Pospelov theory. Then, it seems natural to wonder for the content of Lee-Wick modes in the Myers-Pospelov theory. In this work our main purpose is to rigorously establish these connection.

The layout of this work is the following. In Sec. II we introduce the fermionic Myers and Pospelov model where we found the exact dispersion relation and we compute the corresponding eigenspinor in any background. In Sec. III we quantize the theory and we show explicitly the connection between the Myers-Pospelov model and Lee-Wick theories. In Sec. IV we discuss the origin of violations of microcausality and give the conclusions and final comments.

II. FERMIONIC MYERS-POSPELOV MODEL

The fermionic sector of the Myers-Pospelov theory is given by the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\,\psi + \bar{\psi}\, n\!\!\!/(g_1 + g_2\gamma_5)\,(n\cdot\partial)^2\psi, \tag{1}$$

where g_1 and g_2 are inverse Planck mass dimension couplings constants and n is a dimensionless four vector defining a preferred reference frame with $n^2 = +1, -1, 0$.

Typically, phenomenological studies assume n purely in the temporal direction. However, in this section we will take n as general as possible and eventually we will consider some special choices.

The variation of the Lagrangian (1) produces the equations of motion

$$\left[i\partial\!\!\!/ - m + g_1 n\!\!\!/(n\cdot\partial)^2 + g_2 n\!\!\!/\gamma^5(n\cdot\partial)^2\right]\psi(x) = 0. \tag{2}$$

In momentum space, $\psi(x) = \int d^4p\, e^{-ip\cdot x}\,\psi(p)$, we obtain an algebraic equation,

$$\left(a\!\!\!/ - b\!\!\!/\gamma^5 - m\right)\psi = 0, \tag{3}$$

where $a_\mu \equiv p_\mu - g_1 n_\mu (n\cdot p)^2$ and $b_\mu \equiv g_2 n_\mu (n\cdot p)^2$ are four vectors which will help us to clear up the notation. Also, let us define the following matrices,

$$\hat{\mathcal{E}} \equiv a\!\!\!/ - b\!\!\!/\gamma^5 - m, \quad \hat{h} \equiv \left[a\!\!\!/, b\!\!\!/\right]\gamma^5. \tag{4}$$

The first one, is related to the equation of motion. In fact, the solutions in the momentum space are those which satisfy $\mathcal{E}\psi = 0$, and then they are particular eigenvectors of $\mathcal{E}$. The second quantity is essentially the Pauli-Lubansky vector projected onto the fixed vector n, *i.e.*,

$$\begin{aligned} n\cdot W &= n_\mu \epsilon^{\mu\nu\sigma\rho} p_\nu \mathcal{M}_{\sigma\rho}, \\ &= n_\mu \epsilon^{\mu\nu\sigma\rho} p_\nu \mathcal{S}_{\sigma\rho}, \end{aligned} \tag{5}$$

where $\mathcal{M}$ and $\mathcal{S}$ are the total and spinorial Lorentz generators, respectively. Indeed,

$$\begin{aligned}\hat{h} &= 2\epsilon^{\mu\nu\sigma\rho}a_\mu b_\nu \mathcal{S}_{\sigma\rho},\\ &= 2g_2(n\cdot p)^2\epsilon^{\mu\nu\sigma\rho}p_\mu n_\nu \mathcal{S}_{\sigma\rho},\\ &= -2g_2(n\cdot p)^2 n\cdot W. \end{aligned} \tag{6}$$

To get this relation, we have made use of the fact that, $[\gamma^\mu,\gamma^\nu]\gamma^5 = \frac{i}{2}\epsilon^{\mu\nu\sigma\rho}[\gamma_\sigma,\gamma_\rho] = 2\epsilon^{\mu\nu\sigma\rho}\mathcal{S}_{\sigma\rho}$.

These operators satisfy the interesting relations,

$$\left[\hat{\mathcal{E}},\hat{h}\right] = 0, \quad (\mathcal{E}+2m)\mathcal{E} = a^2 - b^2 - m^2 - \hat{h}. \tag{7}$$

This means that the solutions of the equation of motion can be expressed in terms of the eigenvectors of $\hat{h}$, and that it must be satisfied the relation,

$$a^2 - b^2 - m^2 - h = 0, \tag{8}$$

which provides the general dispersion relation for the Myers-Pospelov theory, in terms of the eigenvalues h. By noticing that,

$$\hat{h}^2 = 4\left[(a\cdot b)^2 - a^2b^2\right]\mathbb{I}, \tag{9}$$

the eigenvalues of $\hat{h}$ are, $h = \pm 2\sqrt{(a\cdot b)^2 - a^2b^2}$. Obviously, this is true for $(a\cdot b)\geq ab$. Otherwise, the operator $\hat{h}$ is antihermitian, and their eigenvalues are purely imaginary with the same absolute value. Hence, for that case, we have a real dispersion relation given by,

$$\begin{aligned}&\Big(p^2 - m^2 - 2g_1(n\cdot p)^3 + n^2(g_1^2-g_2^2)(n\cdot p)^4\Big)^2\\ &\qquad -4(n\cdot p)^4 g_2^2\Big((n\cdot p)^2 - p^2n^2\Big) = 0. \end{aligned} \tag{10}$$

Now, we will calculate the solutions of the equations of motion. As we pointed out above, we can find these solutions among the eigenvectors of $\hat{h}$, $\hat{h}\psi_h = h\psi_h$. Then, let us find those eigenvectors. To do so, we notice that the $\hat{h}$ operator can be written in terms of a rank two antisymmetric tensor, $T_{\mu\nu}\equiv a_\mu b_\nu - a_\nu b_\mu$, that is,

$$\hat{h} \equiv T_{\mu\nu}\epsilon^{\mu\nu\sigma\rho}\mathcal{S}_{\sigma\rho}. \tag{11}$$

From this tensor, we define two orthogonal three-vectors,

$$(\vec{u})^i \equiv T^{0i} = a^0(\vec{b})^i - b^0(\vec{a})^i \equiv u\hat{e}_1^i, \tag{12}$$

$$(\vec{v})^i \equiv \frac{1}{2}\epsilon^{ijk}T_{jk} = (\vec{a}\times\vec{b})^i \equiv v\hat{e}_2^i, \tag{13}$$

and with them

$$(\vec{w})^i \equiv (\vec{u}\times\vec{v})^i \equiv uv\hat{e}_3^i, \tag{14}$$

where $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$ are three orthonormal space vectors on the direction of $\vec{u}$, $\vec{v}$ and $\vec{w}$, respectively. The norm of these vectors are,

$$\begin{aligned}u &= \sqrt{(a^0)^2(\vec{b})^2 + (b^0)^2(\vec{a})^2 - 2(a^0b^0)(\vec{a}\cdot\vec{b})},\\ v &= \sqrt{(\vec{a})^2(\vec{b})^2 - (\vec{a}\cdot\vec{b})^2}. \end{aligned} \tag{15}$$

Note that,

$$\begin{aligned}T^2 &= T_{\mu\nu}T^{\mu\nu} = 2(v^2-u^2),\\ &= 2(a^2b^2 - (a\cdot b)^2) = -\frac{1}{2}h^2. \end{aligned} \tag{16}$$

The negative values of T^2 correspond to real eigenvalues for h and the positive ones correspond to pure imaginary eigenvalues. By making use of the analogy with the electromagnetic tensor F we will call the $T^2<0$ "electric" case and $T^2>0$ the "magnetic" case.

Now, we define the rotations and boosts generators in the spinor representation,

$$\mathcal{J}_i = \frac{1}{2}\epsilon_{ijk}\mathcal{S}^{jk}, \quad \mathcal{K}_i = \mathcal{S}_{0i}, \tag{17}$$

where the spatial indices are referred to the e basis defined above. Then, the $\hat{h}$ operator turns out to be

$$\hat{h} = -4(u\mathcal{J}_1 + v\mathcal{K}_2). \tag{18}$$

Performing a non local boost-like transformation on the eigenspinor in the $\hat{e}_3$ direction

$$\psi_h = e^{-i\eta\mathcal{K}_3}\psi'_h, \tag{19}$$

the $\hat{h}$ operator transforms as

$$\begin{aligned}\hat{h}' &\equiv e^{i\eta\mathcal{K}_3}\hat{h}\,e^{-i\eta\mathcal{K}_3} = -4\left[(u\cosh\eta - v\sinh\eta)\mathcal{J}_1\right.\\ &\qquad \left.+(v\cosh\eta - u\sinh\eta)\mathcal{K}_2\right]. \end{aligned} \tag{20}$$

Because $-1<\tanh\eta<1$, we can distinguish two cases. For $u>v$ we can set $\tanh\eta = \frac{v}{u}$ so that

$$\hat{h}' = -4\sqrt{u^2-v^2}\mathcal{J}_1. \tag{21}$$

However, for $v>u$, we can set $\tanh\eta = \frac{u}{v}$ such that,

$$\hat{h}' = -4\sqrt{v^2-u^2}\mathcal{K}_2. \tag{22}$$

Since the eigenvalues of $\mathcal{J}$ and $\mathcal{K}$ are $\pm\frac{1}{2}$ and $\pm\frac{i}{2}$ respectively, we have $h = \pm 2\sqrt{u^2-v^2}\equiv\epsilon_h|h|$ for $u>v$, and $h=\pm 2i\sqrt{v^2-u^2}\equiv i\epsilon_h|h|$ for $v>u$ as we expected.

The eigenspinors in the chiral representation for $u>v$ can be written as

$$\psi'_h = \begin{pmatrix}\alpha_h\xi_h\\ \beta_h\xi_h\end{pmatrix}, \tag{23}$$

with $(\vec{u}\cdot\vec{\sigma})\xi_h = -\epsilon_h u\xi_h$. Notice that these eigenvectors have the property, in the e basis,

$$\gamma^1\psi_h = \epsilon_h\gamma^0\gamma^5\psi_h. \tag{24}$$

However, for $v>u$, the eigenspinors have the form,

$$\psi'_h = \begin{pmatrix}\gamma_h\chi_h\\ \delta_h\sigma^1\chi_h\end{pmatrix}, \tag{25}$$

with $(\vec{v}\cdot\vec{\sigma})\chi_h = \epsilon_h v \chi_h$. Similarly, the eigenspinors have the property, in the e basis,

$$\gamma^3 \psi'_h = i\epsilon_h \gamma^1\gamma^5 \psi'_h. \tag{26}$$

The constants α_h, β_h, δ_h, γ_h reflects the fact that the eigenspinors are twofold degenerate.

Now, we are ready to find the solutions of the equations of motion in terms of the spinors ψ_h. Performing the same transformation on $\hat{\mathcal{E}}$ we obtain

$$\hat{\mathcal{E}}' \equiv e^{i\eta\mathcal{K}_3}\hat{\mathcal{E}}\, e^{-i\eta\mathcal{K}_3} = \not{a}' - \not{b}'\gamma^5 - m, \tag{27}$$

where, $a'_\mu = \Lambda_\mu^{\ \nu} a_\nu$ and $b'_\mu = \Lambda_\mu^{\ \nu} b_\nu$ with Λ being some functions of the parameter η. The non vanishing components of the four vectors a and b in the e basis are,

$$\begin{aligned} a_1 &= -(\vec{a}\cdot\hat{e}_1) = -\frac{a^0(\vec{b}\cdot\vec{a}) - b^0(\vec{a})^2}{u}, \\ a_3 &= -(\vec{a}\cdot\hat{e}_3) = -a^0\frac{v}{u}, \end{aligned} \tag{28}$$

and,

$$\begin{aligned} b_1 &= -(\vec{b}\cdot\hat{e}_1) = -\frac{a^0(\vec{b})^2 - b^0(\vec{a}\cdot\vec{b})}{u}, \\ b_3 &= -(\vec{b}\cdot\hat{e}_3) = -b^0\frac{v}{u}. \end{aligned} \tag{29}$$

And the transformed components, in terms of the original ones are

$$\begin{aligned} a'_0 &= a_0\cosh\eta + a_3\sinh\eta, \\ &= a_0\left(\cosh\eta - \frac{v}{u}\sinh\eta\right), \\ a'_3 &= a_3\cosh\eta + a_0\sinh\eta, \\ &= -a_0\left(\frac{v}{u}\cosh\eta - \sinh\eta\right), \end{aligned} \tag{30}$$

and,

$$\begin{aligned} b'_0 &= b_0\cosh\eta + b_3\sinh\eta, \\ &= b_0\left(\cosh\eta - \frac{v}{u}\sinh\eta\right), \\ b'_3 &= b_3\cosh\eta + b_0\sinh\eta, \\ &= -b_0\left(\frac{v}{u}\cosh\eta - \sinh\eta\right), \end{aligned} \tag{31}$$

with the components along the directions e_1 and e_2 remaining unchanged.

In the electric case ($u > v$), we have $a'_3 = b'_3 = 0$ and

$$\begin{aligned} a'_0 &= a_0\sqrt{1 - \frac{v^2}{u^2}} = a_0\frac{|h|}{2u}, \\ b'_0 &= b_0\sqrt{1 - \frac{v^2}{u^2}} = b_0\frac{|h|}{2u}. \end{aligned} \tag{32}$$

In the other hand, in the magnetic case ($v > u$), we have $a'_0 = b'_0 = 0$ and,

$$\begin{aligned} a'_3 &= -a_0\sqrt{\frac{v^2}{u^2} - 1} = a_0\frac{|h|}{2u}, \\ b'_3 &= -b_0\sqrt{\frac{v^2}{u^2} - 1} = b_0\frac{|h|}{2u}. \end{aligned} \tag{33}$$

Hence, in the electric case the equations of motion, $\hat{\mathcal{E}}'\psi_h = 0$, are

$$\left[(a'_0 - \epsilon_h b_1)\,\gamma^0 - (b'_0 - \epsilon_h a_1)\,\gamma^0\gamma^5 - m\right]\psi_h = 0, \tag{34}$$

where we have used (24). This equation fixes the constants in the Eq. (23)

$$\begin{aligned} \alpha_h &= \mathcal{N} m, \\ \beta_h &= \mathcal{N}\left[(a'_0 + \epsilon_h b_1) - (b'_0 + \epsilon_h a_1)\right], \end{aligned} \tag{35}$$

where $\mathcal{N}$ is a normalization constant. The equations of motion, $\mathcal{E}'\psi_h = 0$ in the magnetic case are

$$\left[(a_1 - i\epsilon_h b'_3)\gamma^1 - (b_1 - i\epsilon_h a'_3)\gamma^1\gamma^5 - m\right]\psi'_h = 0, \tag{36}$$

where we have used property (26). This implies that the constants in the Eq. (25) are

$$\begin{aligned} \gamma_h &= \mathcal{N}' m, \\ \delta_h &= \mathcal{N}'\left[(b_1 - i\epsilon_h a'_3) - (a_1 - i\epsilon_h b'_3)\right], \end{aligned} \tag{37}$$

where $\mathcal{N}'$ is another normalization constant.

Finally, to get the complete solution to the general Myers-Pospelov equations of motion we must solve for $p^0 = \omega$ as a function of p_1, p_2, p_3 in Eq. (8) or (10), substitute (24), (26), (34) and (35), and do the linear combination,

$$\psi(x) = \sum_{i,h}\int d^3p\, a^i_h(\vec{p})\psi^i_h(\vec{p})e^{i\omega_{ih}(\vec{p})t - i\vec{p}\cdot\vec{x}}, \tag{38}$$

where the index i runs over the solutions $\omega_{ih}(\vec{p})$ for a given h in the Eq. (8), and $a^i_h(\vec{p})$ are integrations constants. To quantize the theory we must promote the constants a to creation and annihilation operators. Some peculiarities arising when $n_0 \neq 0$ in the procedure of quantization will be discussed in the next section. In general, (8) is is a fourth order polynomial in ω for a given value of h, and it would yield at most four real solutions. The negative solutions correspond to antiparticles modes while the positive ones are particles modes.

However, if n has no temporal component the degree of the polynomial in ω will turn to be of order two, and hence we will obtain a total of two solutions for each given value of h. This last situation is similar to the standard case where we have a total of four solutions. The former situation, namely, when $n_0 \neq 0$ we are obtaining double number of solutions than in the standard case. This is due to the fact that we dealing with a theory with higher time derivatives as can be seen from the equation of motion (2). In the next section we will discuss in more detail the nature of this extra solution.

Let us finally make two observations, the first one is that in the magnetic case we will not obtain real solutions for $\omega(\vec{p})$ because $\hat{h}$ has imaginary eigenvalues. The second one is that for n purely timelike $\hat{h}$ is proportional to the helicity and the energy will depend in general on this quantum number. Also we recover the well know results in this case [3, 16].

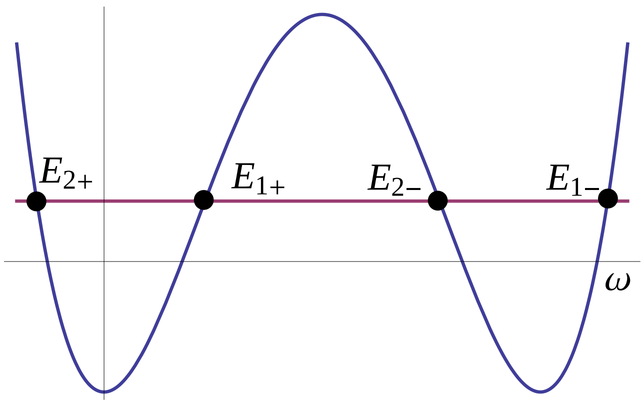

FIG. 1: The curves corresponding to the function $f(\omega)$ and the constant function $g(\omega) = \vec{p}^2$. The intersection of both curves are the solutions of the dispersion relation (39) and $E_{i\pm}$ are given in (52).

III. NON-LOCAL LEE-WICK MODES

In the case n picks up a non vanishing temporal component the physics of the model shows up interesting features connected to the recently revisited model with indefinite metrics [12, 14]. For simplicity in this section let us assume a purely time vector $n = (1, 0, 0, 0)$ and $g_2 = 0$. The dispersion relation (10) reduces to

$$\omega^2 - \vec{p}^2 - m^2 - 2g_1\omega^3 + g_1^2\omega^4 = 0. \tag{39}$$

In order to see the qualitative behavior of the solutions let us define $f(\omega) = \omega^2 - m^2 - 2g_1\omega^3 + g_1^2\omega^4$ and $g(\omega) = \vec{p}^2$, and plot these functions of ω in Fig. 1.

The solutions are those points which intersects the curve f and the horizontal straight line in the upper half plane, which corresponds to the fixed value of the momentum square, *i.e.* $g(\omega) = \vec{p}^2$. Hence, for small values of $\vec{p}$ we find four solutions, one negative frequency which corresponds to an antiparticle and three positive frequencies. Among the positive frequencies the smallest one is the normal particle frequency and the other two correspond to Lee-Wick like modes, as we will show below. It is peculiar the behavior of one the Lee-Wick solutions whose frequency decreases with momentum, this will continue until the momentum reaches the value of $|\vec{p}|_{max} = \sqrt{\frac{1}{16g_1^2} - m^2}$ where it collapses with the normal particle mode. Furthermore, it is worth noting the differences in energy between particles and antiparticles which in the limit $mg_1 << 1$ turns out to be $4\,|g_1|\,m^2$.

Let us discuss how the Lee-Wick modes came about. By looking at the equation of motion for the present case we see that this equation is a second order fermionic equation. Then, let us consider a toy model which displays the essential properties. Let us take the $(0+1)$-dimensional fermion model with higher time derivatives,

$$L_F = -g\bar{\psi}\ddot{\psi} + i\bar{\psi}\dot{\psi} - m\bar{\psi}\psi. \tag{40}$$

The above Lagrangian can be rewritten as

$$\mathcal{L} \;=\; c_+^\dagger (i\partial_t - m_+)\, c_+ - c_-^\dagger (i\partial_t + m_-)\, c_- . \tag{41}$$

In terms of the new variables

$$c_+ = \alpha(i\dot{\psi} + m_-\psi), \quad c_+^\dagger = \alpha(-i\dot{\bar{\psi}} + m_-\bar{\psi}), \tag{42}$$

and,

$$c_- = \alpha(-i\dot{\bar{\psi}} - m_+\bar{\psi}), \quad c_-^\dagger = \alpha(i\dot{\psi} - m_+\psi), \tag{43}$$

where $\alpha = \left(\frac{g}{m_+ + m_-}\right)^{\frac{1}{2}}$ and

$$m_\pm \;\equiv\; \frac{\mp 1 + \sqrt{1 + 4gm}}{2g}, \tag{44}$$

Quantizing this model implies that the Hamiltonian is

$$H_F = m_+ c_+^\dagger c_+ - m_- c_-^\dagger c_- - m_-, \tag{45}$$

with the non vanishing anticommutation relations

$$\{c_+, c_+^\dagger\} = 1, \quad \{c_-, c_-^\dagger\} = -1. \tag{46}$$

This is a four state system with energies $0, m_+, m_-, m_+ + m_-$, and the eigenstates corresponding to the last two energies of negative norm states due to the wrong sign in the anticommutators above.

It is worth noting that the standard first time derivative is reached by performing the limit $mg \to 0$ where m_+ tends to m and m_- blows up to infinity as $1/g$. The generalization to quantum field theory is straightforward and unitarity problems seems to be controlled by suitable definitions of propagators [15].

In our case, recall the Lagrangian

$$L = \int d^3x\, \bar{\psi}(i\partial\!\!\!/ + g_1\gamma^0\, \partial_0^2 - m)\psi, \tag{47}$$

$$= \int d^3x\, \psi^\dagger (i\partial_0 + g_1\, \partial_0^2 - h_D)\psi, \tag{48}$$

where $\hat{h}_D = i\vec{\alpha}\cdot\vec{\nabla} + m\beta$ is the standard Dirac Hamiltonian operator. Now let us write the field in terms of the standard solutions of the Dirac Hamiltonian operator,

$$\psi(x, t) = \sum_{s,i} \int \frac{d^3p}{\sqrt{2E_{0p}}} u_i^s(p)\psi_i^s(p, t)e^{i\vec{p}\cdot\vec{x}}, \tag{49}$$

where s is s spin index, i is the particle and antiparticle index, i.e., $u_1^s = u^s$ and $u_2^s = v^s$ in the standard notation and $E_{0p} = \sqrt{\vec{p}^2 + m^2}$. Remembering that $u_i^{s\dagger}(p)u_j^r = 2E_{0p}\delta^{sr}\delta_{ij}$ and $h_D u_i^s(p)e^{i\vec{p}\cdot\vec{x}} = \epsilon_i E_{0p} u_i^s(p)e^{i\vec{p}\cdot\vec{x}}$ where $\epsilon_1 = +1$ and $\epsilon_2 = -1$, we have

$$L = \sum_{s,i} \int d^3p\, \psi_i^{s\dagger}(p, t)(g_1\partial_0^2 + i\partial_0 + \epsilon_i E_{0p})\psi_i^s(p, t). \tag{50}$$

Now it is clear we have reduced the quantum field theory problem to a set of four quantum mechanical systems at a given momentum. Following our quantum mechanical example each of these systems can be decomposed in terms of two kinds of modes with indefinite metric, that is

$$L = \sum_{s,i} \int d^3p \, \psi^{s\dagger}_{i+} (i\partial_0 - E_{i+}(p)) \psi^s_{i+} \\ - \sum_{s,i} \int d^3p \, \psi^{s\dagger}_{i-} (i\partial_0 - E_{i-}(p)) \psi^s_{i-}, \qquad (51)$$

where

$$E_{i\pm} = (\mp 1 + \sqrt{1 + \epsilon_i 4 E_{0p} g_1}) \frac{1}{2g_1}. \qquad (52)$$

It is worth noting that the fields $\psi^s_{i\pm}$ are non local in the sense that they are defined

$$\psi^s_{i\pm} = (i\partial_0 - \hat{E}_{i\mp}) \psi^s_{i\pm}, \qquad (53)$$

where $\hat{E}_{i\pm}$ are non local operators depending on the spatial derivatives. As in our quantum mechanical example the plus modes are standard particles with positive norm and the minus modes are Lee-Wick particles with negative norm. Both modes have the degeneracy of spin and then we get a total of eight modes. There dispersion relation are those in (52) which agree with our qualitative argument in the beginning of the section.

IV. DISCUSSIONS AND CONCLUSIONS

In this work we have studied the general solution of the fermion Myers and Pospelov model. We have discussed and analyzed the appearance of non local Lee-Wick modes, giving the prescription to quantize the theory.

To conclude we would like to discuss microcausality in this model. Firstly, the fact that we are dealing with an explicitly Lorentz invariance violating model, we expect a small failure of microcausality at the kinematical level [10, 17, 18]. This can be checked by computing the first order corrections of the anticommutator of free fermionic fields at different spacetime points,

$$\{\psi(x), \bar{\psi}(0)\} = \hat{\mathcal{O}} \oint_C \frac{d^4p \quad e^{-ip\cdot x}}{\Big(a^2(p) - b^2(p) - m^2\Big)^2 - h^2(p)}, \qquad (54)$$

where $\hat{\mathcal{O}}$ is a local differential operator and C is a closed curve enclosing all the poles. However, there is another source of microcausality violation which come from the Lee-Wick nature of the model when the temporal component of n_μ is not zero [12]. Even though this cannot been seen from the anticommutator of the free fermions above we will obtain modifications after adding interactions to the theory. Then, the origin of the Lee-Wick failure of microcausality has a dynamical character.

It would be interesting to study how this two sources of microcausality failure can interfere with each other. Finally we would like to mention that most of the results in this work could be generalized to the remaining sectors of the Myers-Pospelov theory.

Acknowledgments

J.L. acknowledges support from DICYT Grant No. 041131LS (USACH) and FONDECYT-Chile Grant No. 1100777. C.M.R. acknowledges partial support from DICYT (USACH) and Dirección de Investigación de la Universidad del Bío Bío (DIUBB).

[1] D. Colladay and V. A. Kostelecky, Phys. Rev. D**55**, 6760 (1997); D. Colladay and V. A. Kostelecky, Phys. Rev. D**58**, 116002 (1998); V. A. Kostelecky, M. Mewes, Phys. Rev. **D80**, 015020 (2009).
[2] V. A. Kostelecky, N. Russell, Rev. Mod. Phys. **83**, 11 (2011); J. -P. Bocquet, D. Moricciani, V. Bellini, M. Beretta, L. Casano, A. D'Angelo, R. Di Salvo, A. Fantini *et al.*, Phys. Rev. Lett. **104**, 241601 (2010).
[3] R. C. Myers and M. Pospelov, Phys. Rev. Lett. **90** (2003) 211601.
[4] P. A. Bolokhov and M. Pospelov, Phys. Rev. **D 77**, 025022 (2008).
[5] R. Montemayor and L. F. Urrutia, Phys. Rev. D **72**, 045018 (2005); R. Montemayor and L. F. Urrutia, Phys. Lett. **B 606** (2005) 86.
[6] L. Maccione, S. Liberati, A. Celotti and J. G. Kirk, JCAP **0710**, (2007) 013; L. Maccione and S. Liberati, JCAP **0808**, (2008) 027.
[7] J. R. Ellis, N. Harries, A. Meregaglia, A. Rubbia, A. Sakharov, Phys. Rev. **D78**, 033013 (2008); E. Di Grezia, S. Esposito, G. Salesi, Mod. Phys. Lett. **A21**, 349-362 (2006).
[8] C. M. Reyes, L. F. Urrutia, J. D. Vergara, Phys. Rev. **D78**, 125011 (2008); C. M. Reyes, L. F. Urrutia, J. D. Vergara, Phys. Lett. **B675**, 336-339 (2009); T. Mariz, Phys. Rev. **D83**, 045018 (2011).
[9] G. Gubitosi, G. Genovese, G. Amelino-Camelia and A. Melchiorri, Phys. Rev. **D82**, 024013 (2010).
[10] C. M. Reyes, Phys. Rev. D **82**, 125036 (2010).
[11] B. Altschul, Phys. Rev. **D83**, 056012 (2011); J. Gamboa, J. Lopez-Sarrion, A. P. Polychronakos, Phys. Lett. **B634**, 471-473 (2006).
[12] J. R. Espinosa, B. Grinstein, D. O'Connell, M. B. Wise, Phys. Rev. **D77**, 085002 (2008); B. Grinstein,

D. O'Connell, M. B. Wise, Phys. Rev. **D77**, 025012 (2008).
[13] M. Ostrogradski, Mem. Acad. St. -Pétersbourg **VI**, 385 (1850), A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**, 145 (1950).
[14] T. D. Lee, G. C. Wick, Phys. Rev. **D2**, 1033-1048 (1970).
[15] R. E. Cutkosky, P. V. Landshoff, D. I. Olive, J. C. Polkinghorne, Nucl. Phys. **B12**, 281-300 (1969).
[16] O. Bertolami, J. G. Rosa, Phys. Rev. **D71**, 097901 (2005).
[17] V. A. Kostelecky, R. Lehnert, Phys. Rev. **D63**, 065008 (2001).
[18] C. Adam, F. R. Klinkhamer, Phys. Lett. **B513**, 245-250 (2001); C. Adam, F. R. Klinkhamer, Nucl. Phys. **B607**, 247-267 (2001).